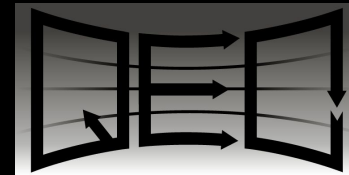


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# Exponentially-Biased Ground-State Sampling of Quantum Annealing Machines with Transverse-Field Driving Hamiltonians

Dr. Salvatore Mandrà

# What is fair sampling?

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## Definition (fair sampling):

- The ability of an algorithm to find all solutions of a degenerate problem with equal probability when run in **repetition mode**

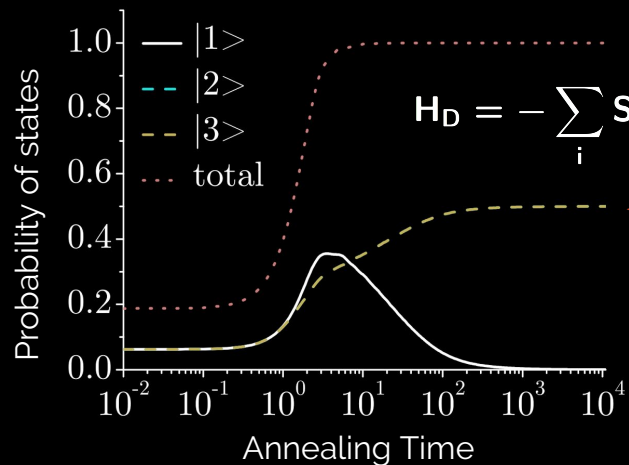
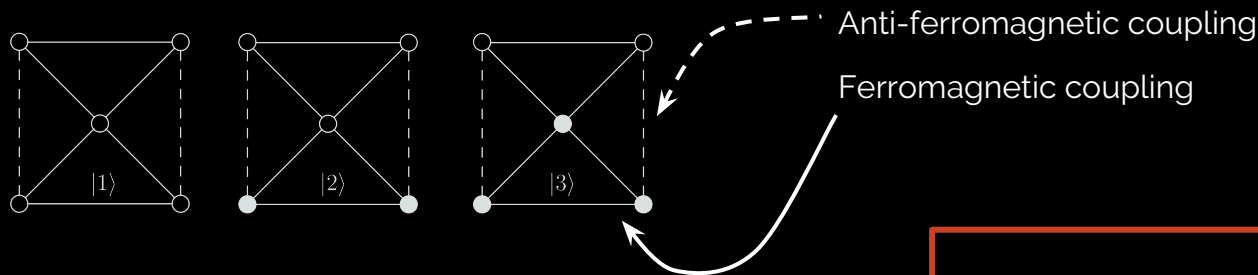
## Why is it important?

- In some contexts (SAT-Filter, #SAT, machine learning, ...) finding a **good variety** of solutions is more important than finding a single solution quickly

## Optimize benchmarking:

- Standard test: Find the ground-state energy **fast and reliably**
- Stringent test: Find **all minimizing configurations** equiprobably

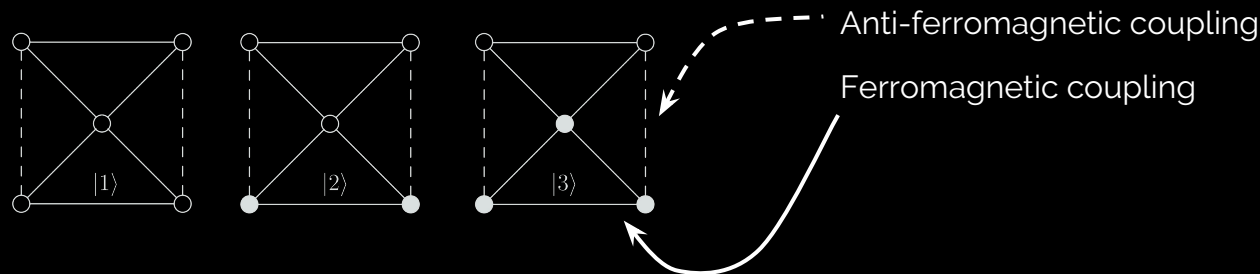
# Previous studies on transverse field QA [1]



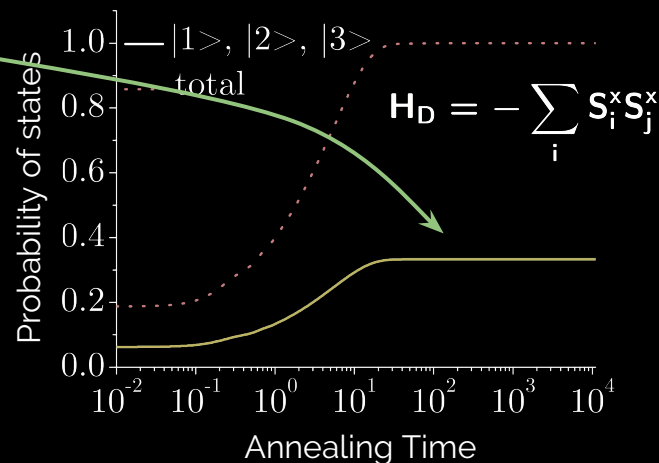
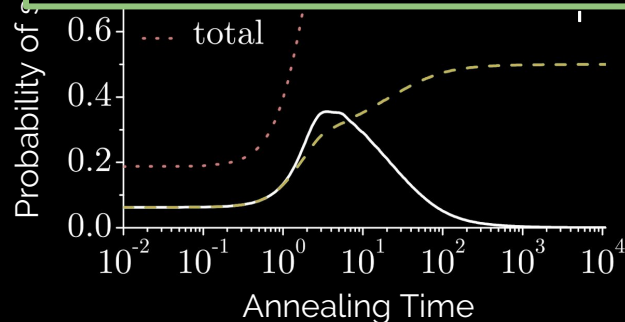
Transverse field QA  
is **biased** ...

$$H_D = - \sum_i S_i^x$$

# Previous studies on transverse field QA [1]

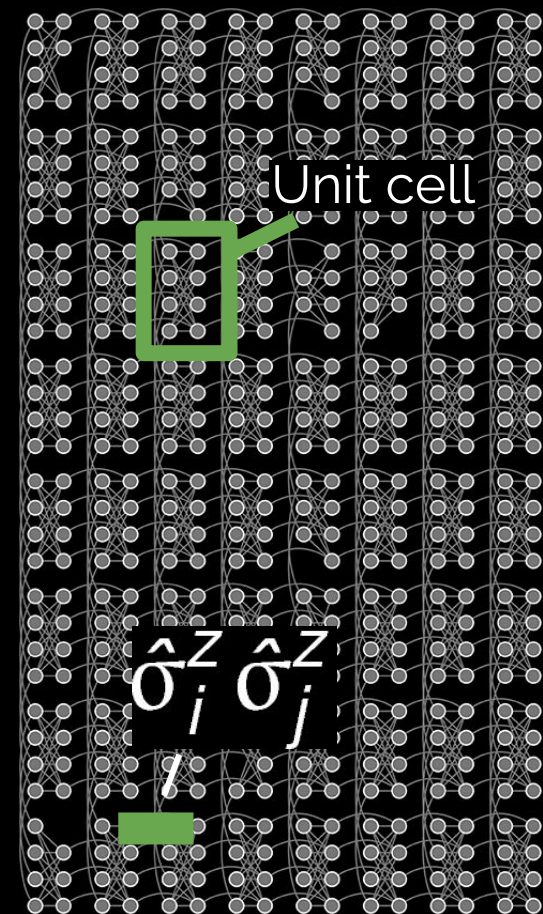


**Non-stoquastic  $H_D$  mitigates the problem!**



[1] Y. Matsuda, H. Nishimori & H. G Katzgraber, "Ground-state statistics from annealing algorithms: quantum versus classical approaches.", New Journal of Physics, 11(7), 073021 (2009)

# The D-Wave 2X quantum annealer



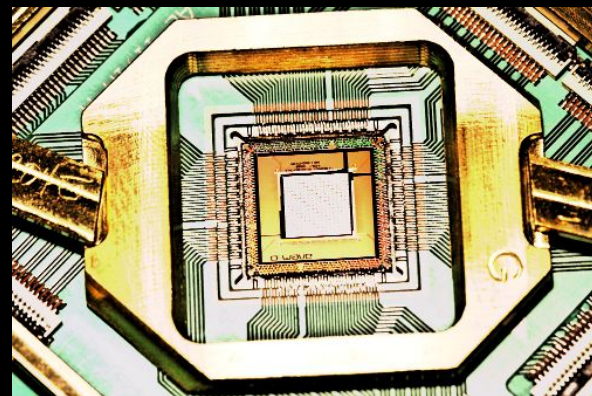
—  $H_p$

$$H_D = - \sum_i \hat{\sigma}_i^x$$

- Unavoidable **noise**
- Non-zero **temperature**

~1000 working qubits

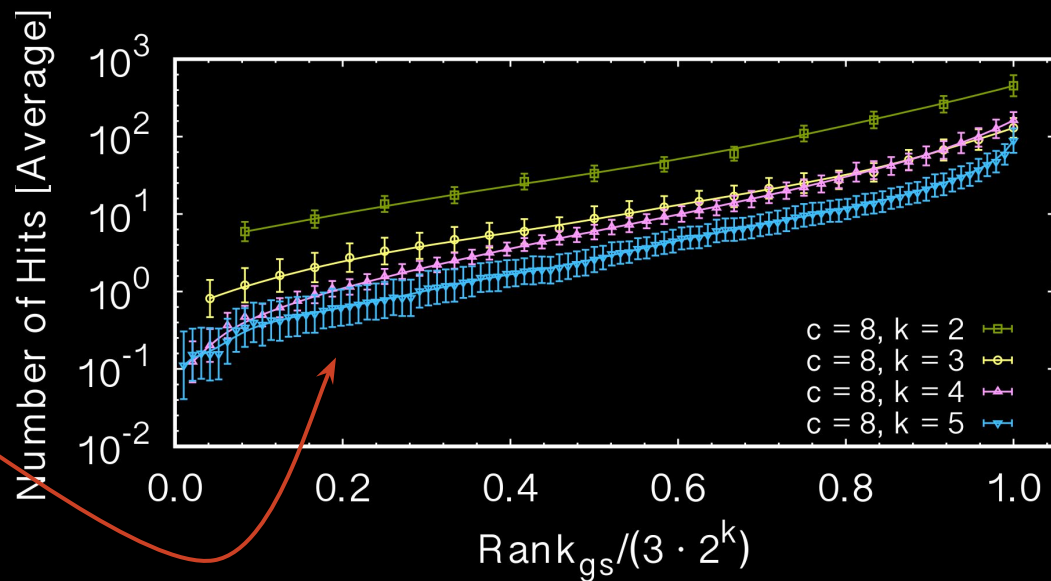
Superconducting qubit chip



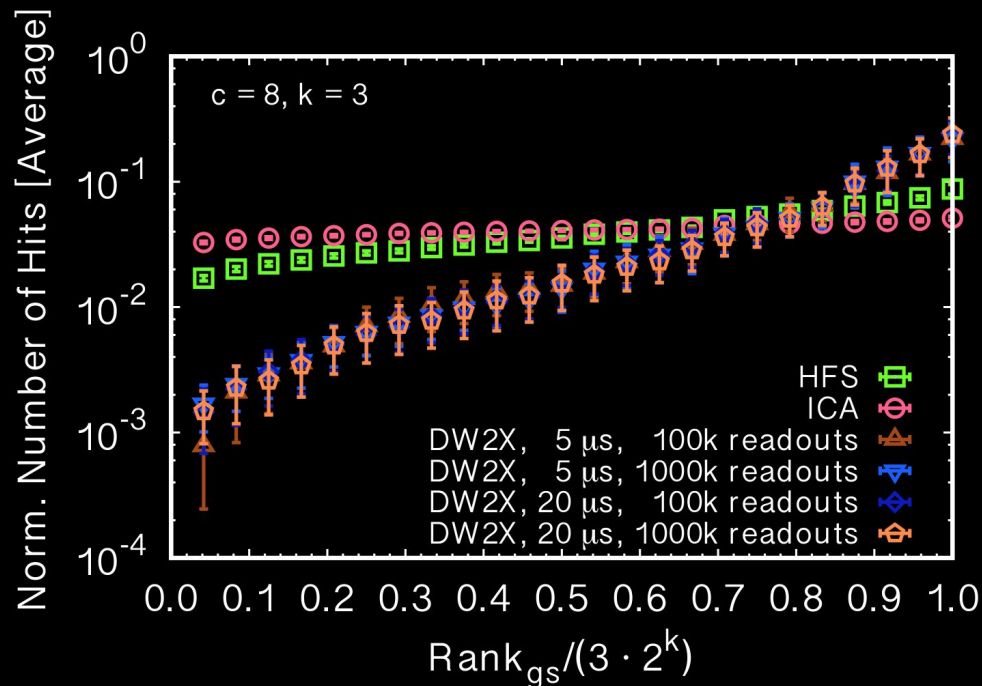
# Experimental analysis using DW2X device [1]

- Random couplings from **Sidon set** ( $J_{ij} = \pm 5, \pm 6, \pm 7$  on Chimera of  $c \times c$  unit cells)
- Limit the study to instances with **well controlled degeneracy** ( $\#_{gs} = 3 \cdot 2^k$ )
- No **trivial** degeneracy
- 100 gauges x {10k, 100k} readouts
- $T_{ann} = 5\mu, 20\mu, 200\mu$

**DW2X is  
exponentially biased!**

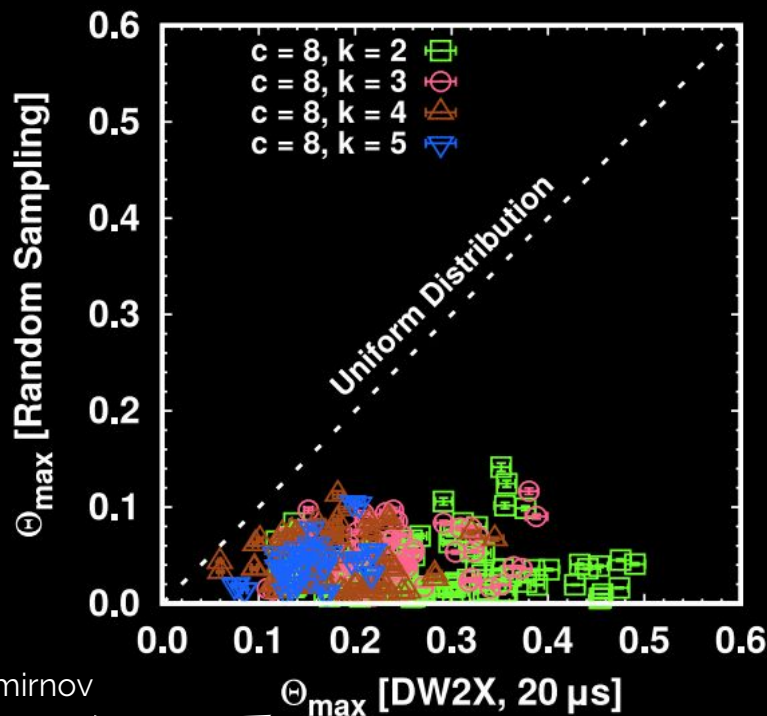


# Classical algorithms sample more homogeneously

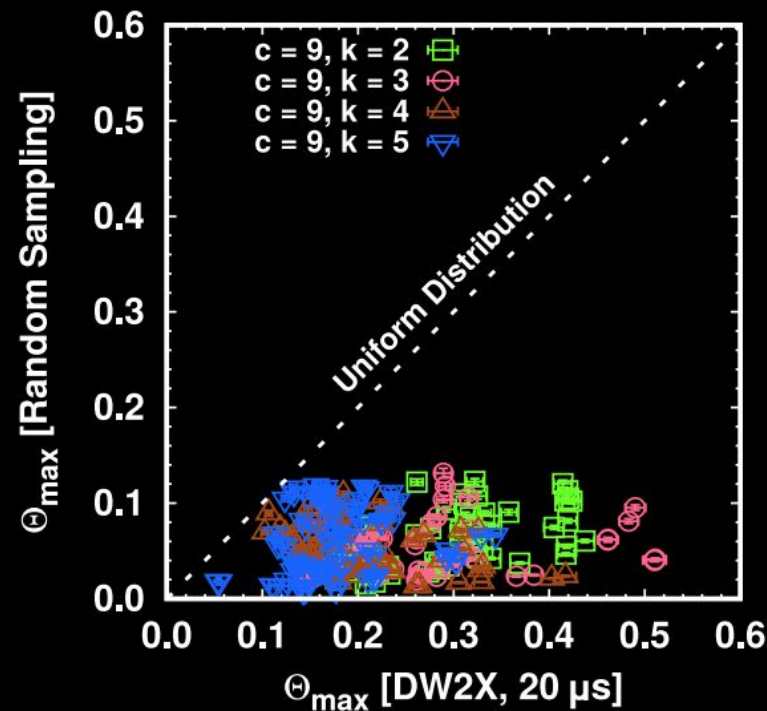


- [1] **S. Mandrà**, Z. Zhu & H. G. Katzgraber, "Exponentially-Biased Ground-State Sampling of Quantum Annealing Machines with Transverse-Field Driving Hamiltonians", arXiv:1606.07146
- [2] F. Hamze & N. de Freitas, Proceedings (2004), A. Selby, arXiv (2014)
- [3] Z. Zhu, A. J. Ochoa & H. G. Katzgraber, PRL (2015)

# Experimental analysis using DW2X device [1]



Kolmogorov-Smirnov  
Test



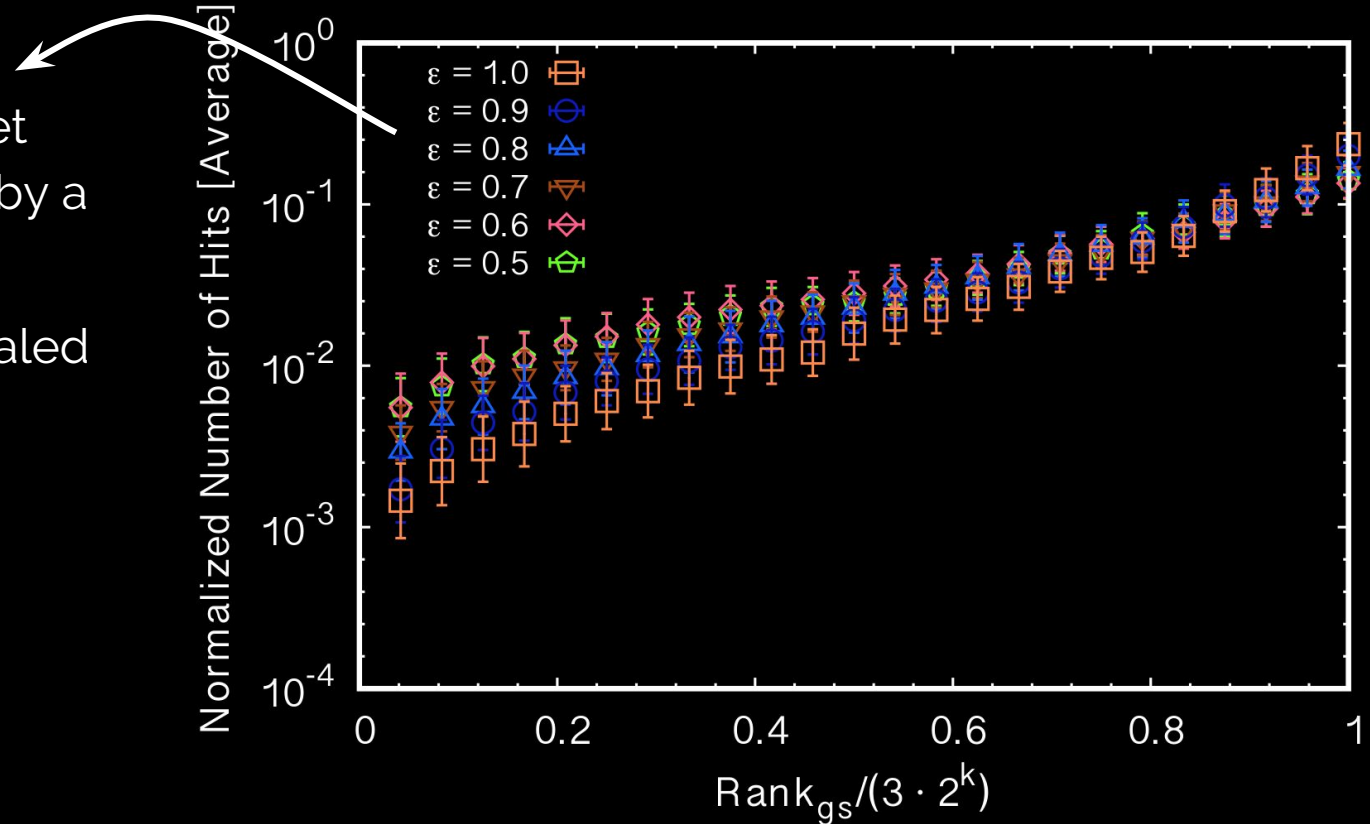


Could the **bias** be a consequence of  
the **intrinsic noise** of the DW2x?

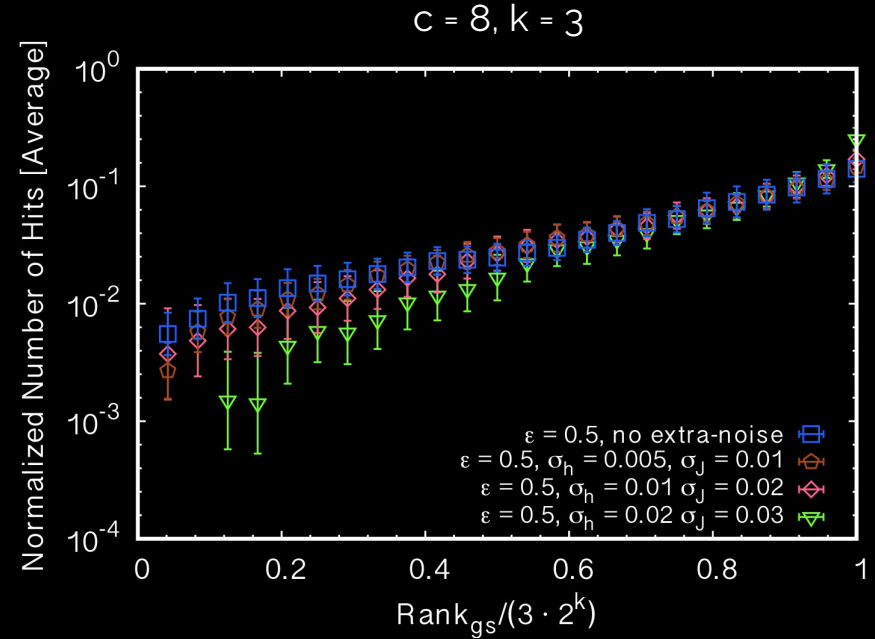
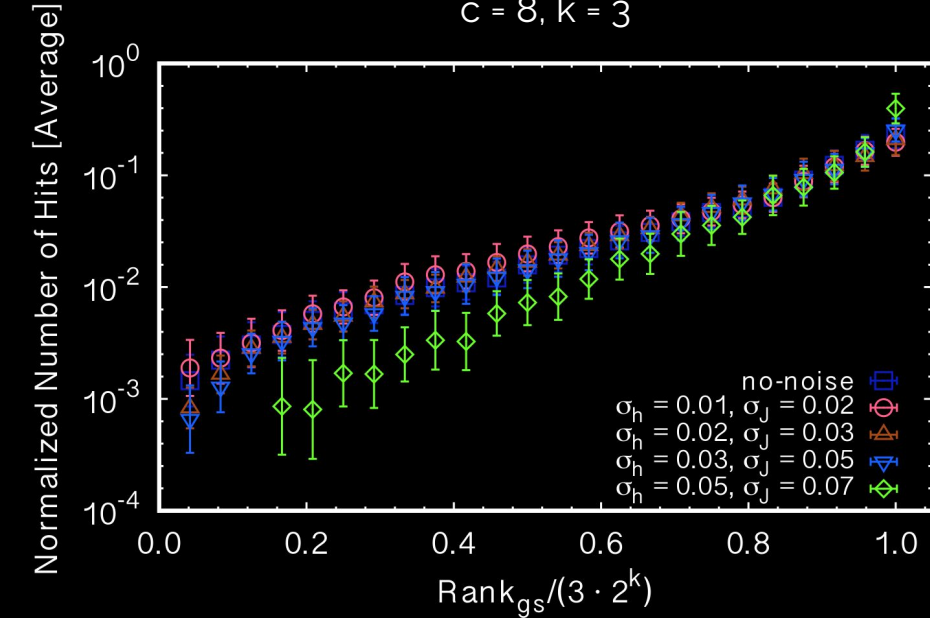
**No.**

# The bias is **unchanged** by rescaling the energy

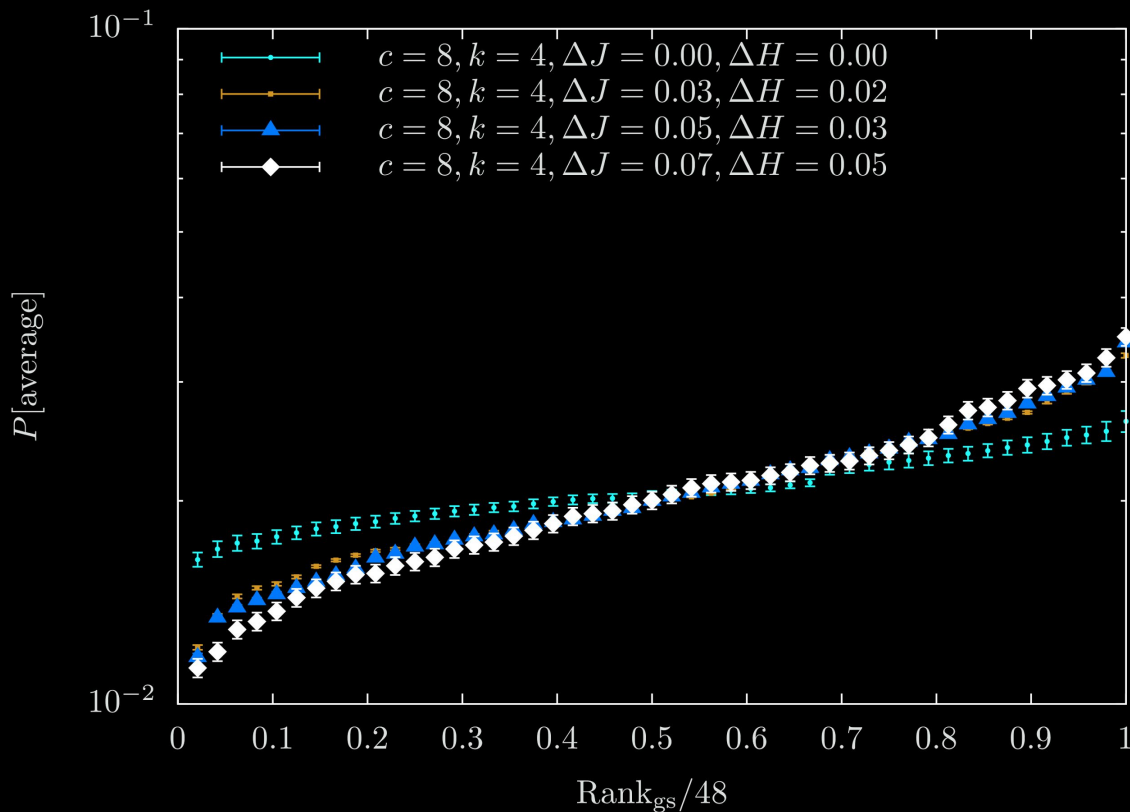
- Energy of the target problem rescaled by a factor  $\varepsilon$
- Intrinsic noise rescaled by a factor  $1/\varepsilon$



# Adding extra noise does not change the bias

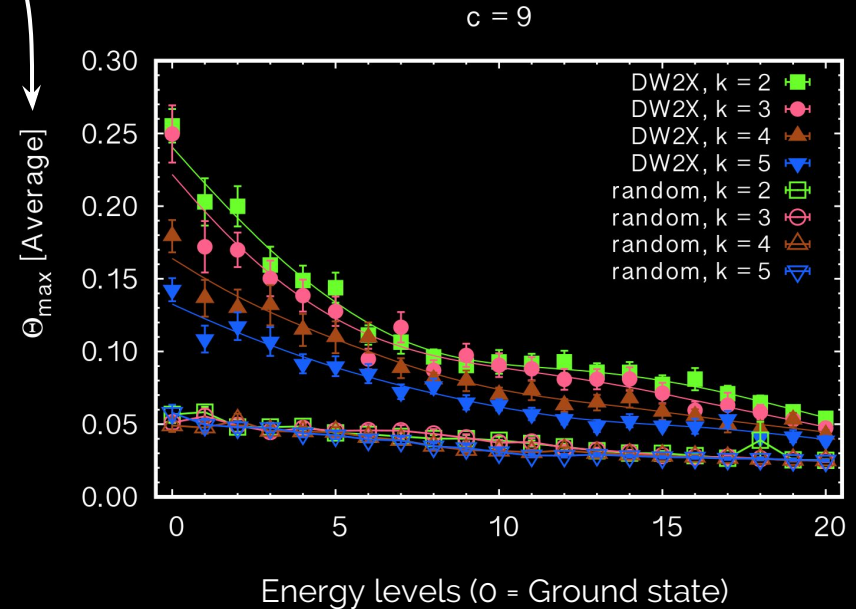
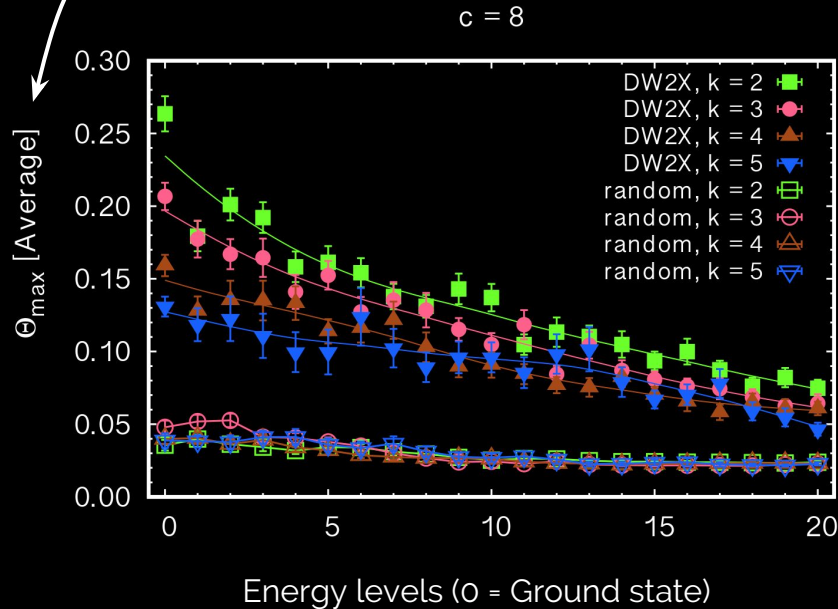


# Classical algorithms are marginally affected by the noise



# The bias persists up to the 20<sup>th</sup> excited state!

Different of the sampling respect to the flat distribution (**larger is worse**)



# Implications & Future directions

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## The bias can limit the use of QA for sampling

- Applications like SAT-Filter and machine learning may not be suitable for QA without mitigating the sampling problem

## How to mitigate the sampling problem?

- Explore different driver Hamiltonians (e.g. non-stoquastic)

## How to understand the bias problem better?

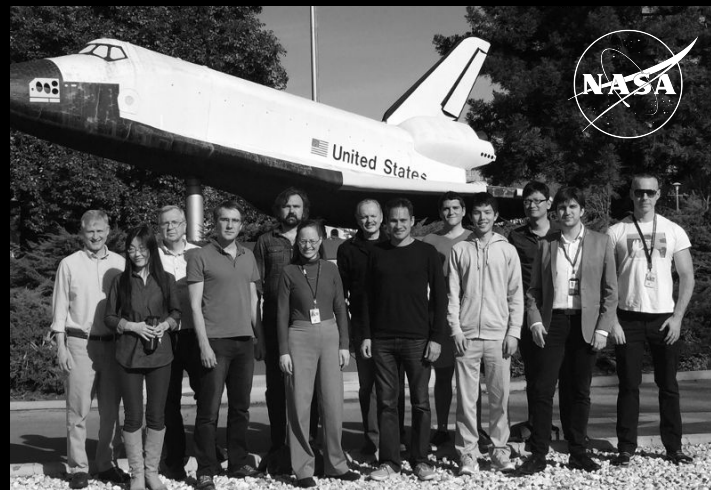
- Theoretical understanding of the role of the driver Hamiltonian in sampling
- Theoretical exploration of the implication of many-body localization



**Zheng Zhu**  
**Texas A&M**



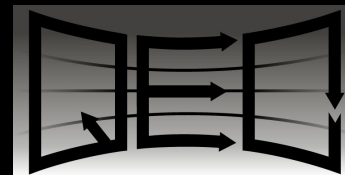
**Helmut G.  
Katzgraber**  
**Texas A&M**



**NASA QUAIL**



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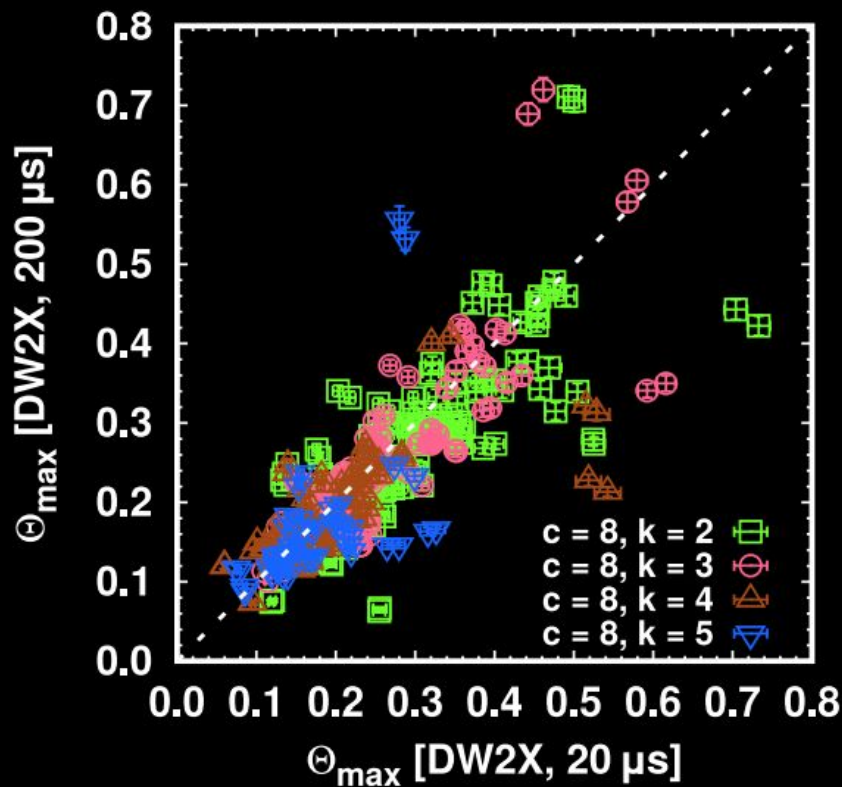


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**Thanks for the attention!**

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# Experimental analysis using DW2X device [1]



[1] **S. Mandrà**, Z. Zhu & H. G. Katzgraber, "Exponentially-Biased Ground-State Sampling of Quantum Annealing Machines with Transverse-Field Driving Hamiltonians", arXiv:1606.07146



# Adiabatic Quantum Optimization (AQO)

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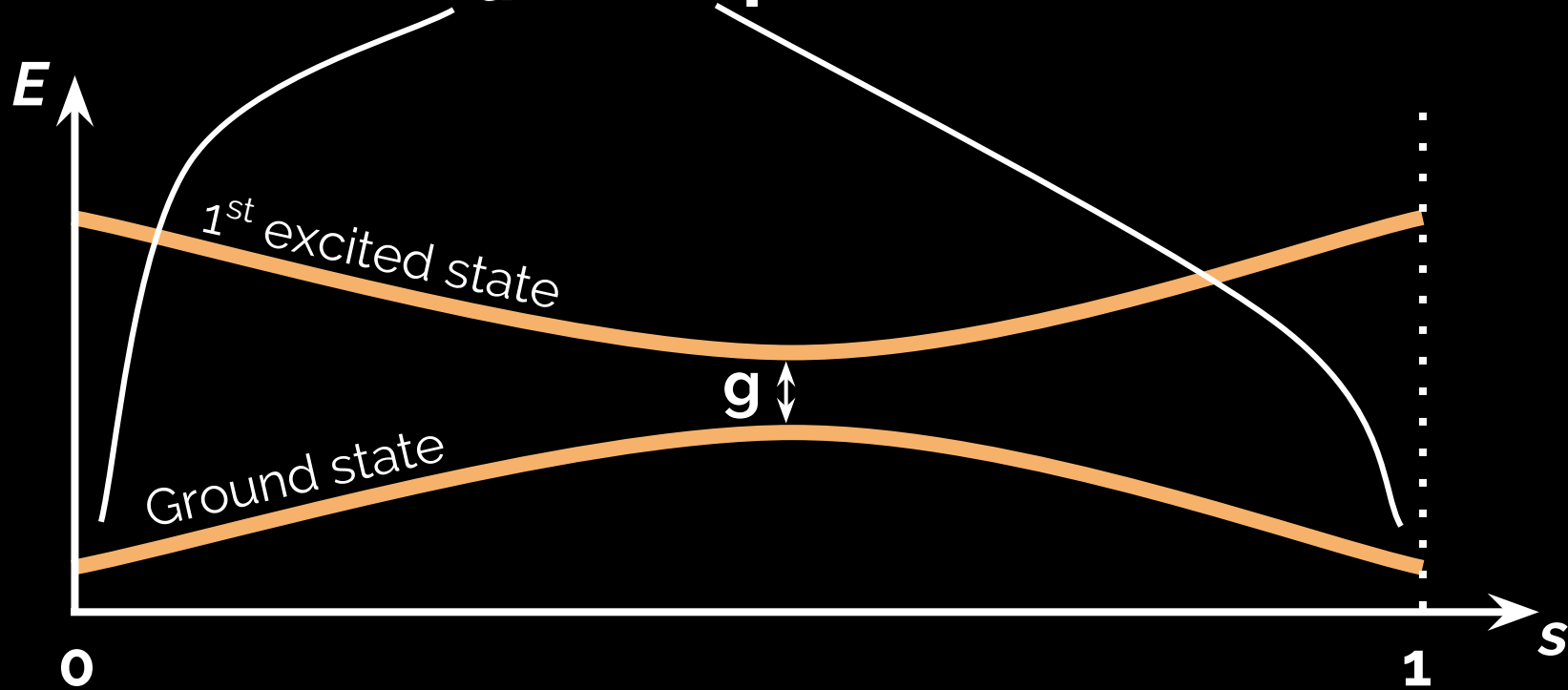
$$\mathbf{H} = (\mathbf{1} - \mathbf{s})\mathbf{H}_d + \mathbf{s}\mathbf{H}_p$$

Initial “driver”  
Hamiltonian

Target Problem

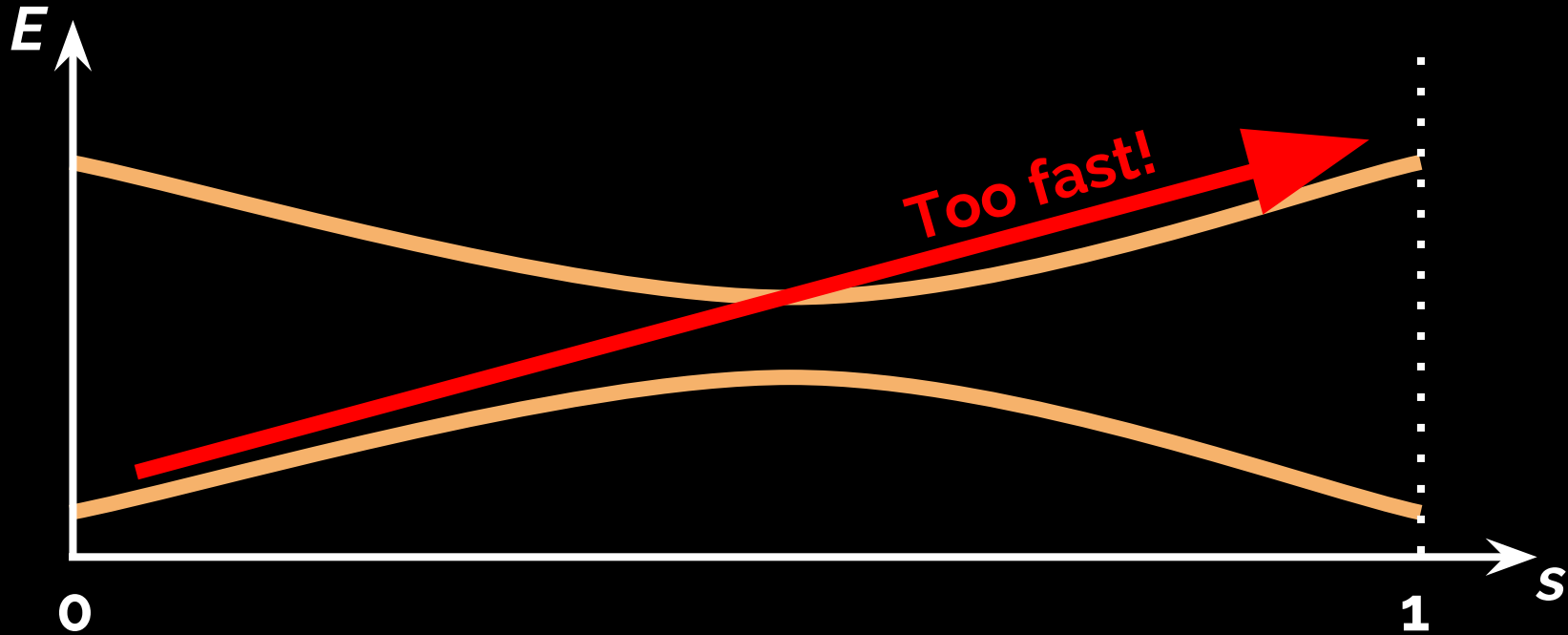
# Adiabatic Quantum Optimization (AQO)

$$H = (1 - s)H_d + sH_p$$



# Adiabatic Quantum Optimization (AQO)

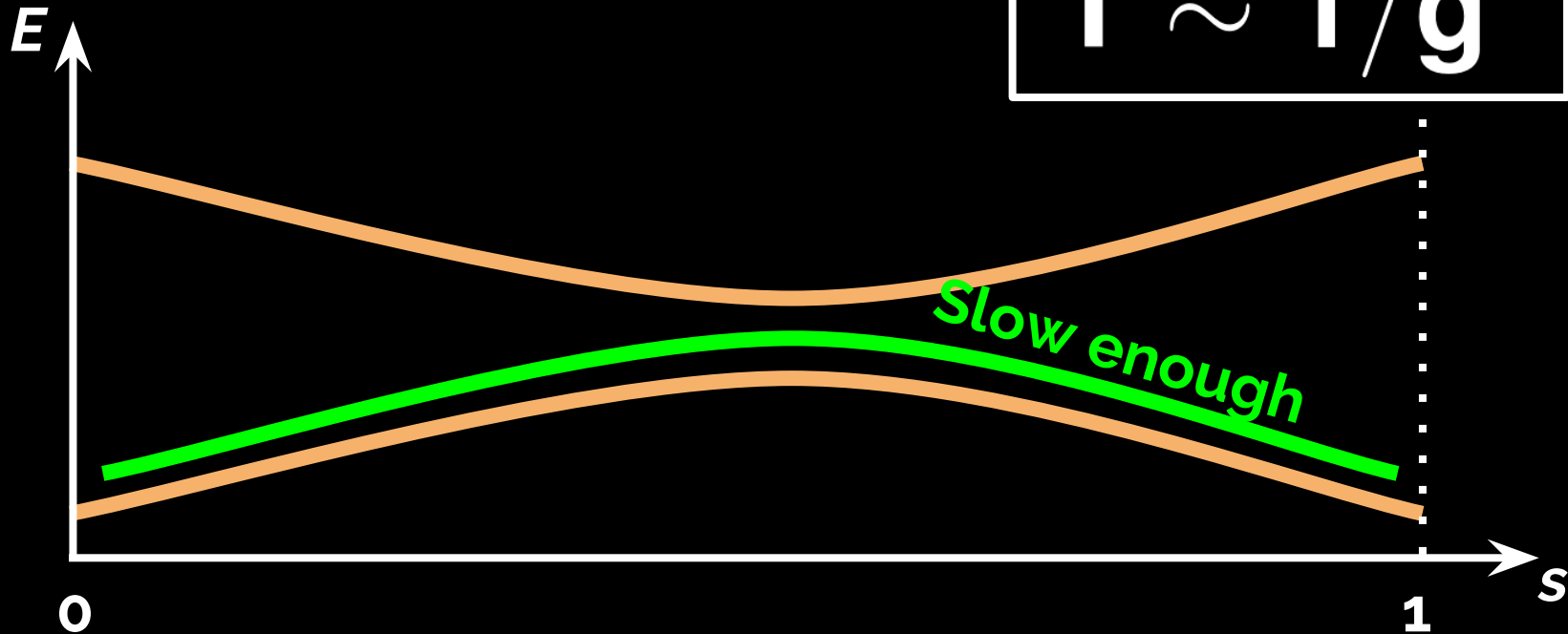
$$H = (1 - s)H_d + sH_p$$



# Adiabatic Quantum Optimization (AQO)

$$H = (1 - s)H_d + sH_p$$

$$T \sim 1/g^2$$



# Adiabatic Quantum Optimization (AQO)

$$H = (1 - s)H_d + sH_p$$

